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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA,

357. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

Solve the system

$$\begin{aligned}\sqrt{x^2+a^2+b^2+c^2} &= \sqrt{y^2+b^2+c^2} + \sqrt{z^2+b^2+c^2}, \\ \sqrt{y^2+a^2+b^2+c^2} &= \sqrt{x^2+a^2+c^2} + \sqrt{z^2+a^2+c^2}, \\ \sqrt{z^2+a^2+b^2+c^2} &= \sqrt{x^2+a^2+b^2} + \sqrt{y^2+a^2+b^2}.\end{aligned}$$

Solution by B. F. FINKEL, Ph. D., Drury College.

Transposing the first term of the second member of the first equation, squaring, collecting, and transposing, we have

$$x^2+y^2-z^2+s^2=2\sqrt{(x^2+s^2)}\sqrt{(y^2+s^2-a^2)}\dots(1),$$

where $s^2=a^2+b^2+c^2$. Similarly, we obtain from the second equation,

$$x^2+y^2-z^2+s^2=2\sqrt{(y^2+s^2)}\sqrt{(x^2+s^2-b^2)}\dots(2),$$

and from the third,

$$x^2+z^2-y^2+s^2=2\sqrt{(z^2+s^2)}\sqrt{(x^2+s^2-c^2)}\dots(3).$$

By transposing the second term of the first equation, squaring, collecting, and retransposing, we get

$$x^2+z^2-y^2+s^2=2\sqrt{(x^2+s^2)}\sqrt{(z^2+s^2-a^2)}\dots(4).$$

From (1) and (2) we have,

$$\sqrt{(x^2+s^2)}\sqrt{(y^2+s^2-a^2)}=\sqrt{(y^2+s^2)}\sqrt{(x^2+s^2-b^2)};$$

whence, by squaring and dividing by $(x^2+s^2)(y^2+s^2)$, we get

$$\frac{y^2+s^2-a^2}{y^2+s^2}=\frac{x^2+s^2-b^2}{x^2+s^2}.$$

Hence, $\frac{a^2}{y^2+s^2} = \frac{b^2}{x^2+s^2}$, or $y^2+s^2 = \frac{a^2}{b^2}(x^2+s^2)$.

From (3) and (4), we obtain, in like manner,

$$z^2+s^2 = \frac{a^2}{c^2}(x^2+s^2).$$

Substituting these values of z^2+s^2 and y^2+s^2 in the first equation, we have

$$\sqrt{(x^2+s^2)} = \sqrt{\frac{a^2}{b^2}(x^2+s^2)-a^2} + \sqrt{\frac{a^2}{c^2}(x^2+s^2)-a^2};$$

$$\text{whence, } bc\sqrt{(x^2+s^2)} = ab\sqrt{(x^2+s^2-b^2)} + ab\sqrt{(x^2+s^2-c^2)}.$$

Squaring,

$$b^2c^2(x^2+s^2) = a^2c^2(x^2+s^2-b^2) + 2a^2bc\sqrt{(x^2+s^2-b^2)}\sqrt{(x^2+s^2-c^2)} + a^2b^2(x^2+s^2-c^2);$$

transposing, and combining,

$$(b^2c^2 - a^2c^2 - a^2b^2)(x^2+s^2) + 2a^2b^2c^2 = 2a^2b^2c^2\sqrt{(x^2+s^2-b^2)}\sqrt{(x^2+s^2-c^2)}.$$

Squaring both members of this equation, and rearranging terms, we have

$$[(b^2c^2 - a^2c^2 - a^2b^2)^2 - 4a^4b^2c^2](x^2+s^2) + 4a^2b^4c^4(x^2+s^2) = 0.$$

$$\therefore x^2+s^2=0, \text{ or } x^2+s^2=$$

$$\frac{4a^2b^4c^4}{4a^4b^2c^2 - (b^2c^2 - a^2c^2 - a^2b^2)^2}$$

$$= \frac{4a^2b^4c^4}{(b^2c^2 - a^2c^2 - a^2b^2 + 2a^2bc)(-b^2c^2 + a^2c^2 + a^2b^2 + 2a^2bc)}$$

$$= \frac{4a^2b^4c^4}{[b^2c^2 - a^2(b-c)^2][a^2(b+c)^2 - b^2c^2]}$$

$$= \frac{4a^2b^4c^4}{(bc+ab-ac)(bc-ab+ac)(ab+ac+bc)(ab+ac-bc)}$$

$= \frac{4a^2b^4c^4}{\Delta}$, where Δ is the denominator of the above fraction.

$$y^2+s^2 = \frac{4a^4b^2c^4}{\Delta}, \text{ and } z^2+s^2 = \frac{4a^4b^4c^2}{\Delta}.$$

$$\therefore x = \pm \left(\frac{4a^4b^2c^4 - s^2 \Delta}{\Delta} \right)^{\frac{1}{2}}, \quad y = \pm \left(\frac{4a^4b^2c^4 - s^2 \Delta}{\Delta} \right)^{\frac{1}{2}}, \text{ and}$$

$$z = \pm \left(\frac{4a^4b^4c^2 - s^2 \Delta}{\Delta} \right)^{\frac{1}{2}}.$$

$x^2+s^2=0$ is not admissible.

PROBLEMS FOR SOLUTION.

ALGEBRA.

363. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

(a) If a and n be positive integers, the integral part of $[a + \sqrt{(a^2-1)}]^n$ is odd.

(b) If a and n be positive integers, the integral part of $[\sqrt{(a^2+1)} + a]^n$ is odd when n is even and even when n is odd. [From Todhunter's *Algebra*, p. 353].

364. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The English physicist, Hooke, published the discovery contained in the Latin sentence, "Ut tensio sic vis" by the cypher *cciiinosssttuv*. Preserving the lexicographical order, find which permutation, taking all letters, the Latin sentence is from the cypher.

365. Proposed by C. N. SCHMALL, New York City.

In still water, a steam tug goes 6 miles an hour less when towing a barge than when alone. Having drawn the barge 30 miles up a stream, whose current runs 1 mile an hour, it returns alone and completes the journey in $12 \frac{8}{11}$ hours. Find the rate of the tug in still water.

GEOMETRY.

396. Proposed by DANIEL KRETH, Oxford, Iowa.

In the triangle ABC , $AB=214$, $BC=263$, and $AC=405$. A point P is situated in the same horizontal plane; angle $BPA=13^\circ 30'$ and angle $BPC=29^\circ 50'$. Find the distances, AP , BP , and CP .